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# Measurement: Logs and pH

**URL:** [**http://mathbench.umd.edu/modules-au/measurement\_logs/page01.htm**](http://mathbench.umd.edu/modules-au/measurement_logs/page01.htm)

## Learning Outcomes

After completing this module you should be able to:

* Work with very large and very small numbers by converting between log and linear scales, using logarithms to the base 10 and anti-logarithms
* Use the log scale of hydrogen ion concentration (pH) to determine the acidity of the solution
* Determine the hydrogen ion concentration of a solution from its pH

## When size matters

As you may have noticed, scientists like to measure things, but lots of things are too big to get their hands on, and lots of things are too small. For example:

|  |  |
| --- | --- |
| too big | too small |
| rainforests  planets  solar systems  galaxies  universes | grains of pollen  blood cells  *E. coli*  enzymes  carbon atoms  electrons |

What ends up happening is that we need a lot of zeros to express these sizes. For example:

**The number of stars in our galaxy is roughly 100,000,000,000**

**The distance from here to the edge of the universe is roughly  
94,608,000,000,000,000,000,000 km**

The problem is just as bad when we talk about small things:

**The diameter of a red blood cell is 0.000007 m   
The mass of a carbon atom is 0.0000000000000000000000199 grams**

Obviously these are numbers that are hard to deal with. A red blood cell with a diameter of 0.00007 m instead of 0.000007 m would be 10 times too big to fit through a capillary, yet it's hard to tell the numbers apart by looking at them – and hard to remember how many zeros there are supposed to be in the first place!

Of course **one solution to this problem is to use specialised units**. For example, we usually talk about the size of the universe in light-years, while we express the size of a red blood cell in micrometres. **However, these different units make it hard to compare the sizes of things**.

## There must be a better way!

Indeed there must. Scientists often use an alternative numbering system called a “log” scale. **When you put a number on a log scale, you are basically saying “how many zeros are in that number?”**

|  |  |
| --- | --- |
| log(10000)=5  log size 5 needed -- to hold 5 zeros.  Log(10,000) = 5 | log(1,000,000,000)=9  log size 9 needed -- to hold 9 zeros.  Log(1,000,000,000) = 9 |

## ****A number like 10000 has zeros "taking up space" before the decimal point, while a number like 0.0001 has zeros “taking up space" after the decimal point****. In a sense, this space taken up is in the "opposite" direction. We show this by making the log negative rather than positive. So,

## ****log(10000)**** = 4 (count the zeros after the first digit)

## ****log(0.0001)**** = -4 (count the decimal point and the zeros after it, but stop when you get to a non-zero digit)

## So big numbers produce positive logs and small numbers produce negative logs. There must also be a log that is exactly 0, right? So what is the balancing point between big and small numbers?

## balance-rollover

## ****SOME FINE PRINT THAT YOU STILL SHOULDN'T SKIP:**** This module talks about log base 10 which is written log 10 or just log-- there are other bases, notably log base 2 (log 2 ) used in computers, and the natural log (loge or ln) used in any science that deals with growth or decay. However, log base 10 is a common base and arguably the easiest to understand. ****When using your calculator****, you may have keys labeled "LOG" (for log base-10) and "LN" (for natural log). You want to use the log key, not the LN key. When in doubt, check that log(10) = 1. END OF FINE PRINT

## How about an example?

## ****The stars in our galaxy:**** a hundred billion is a 1 with 11 zeros after it, so the log scale version of this number is simply 11.

## ****Your chances of winning the lottery:**** let's say it's one-in-a-million. Expressed as a decimal, that is 0.000001. So, count the decimal place and the zeros AFTER it, and stop when you get to the non-zero digit:

## ****log (0.000001) = -6****

## Special Cases

## We know that

## ****log (1000) = 3, and****

## ****log (0.001) = -3****

## What about a number that has both a "large" part before the decimal and a "small" part after the decimal? For example

## ****log (1000.001) = ???****

## Remember, the whole purpose of logs is to tell you approximately how big the number is. Therefore, the decimal part (0.001) is really pretty unimportant compared to the whole-number part (1000). So, the log of this number is pretty close to the log of 1000, plus a tiny bit thrown in for the 0.001. In fact, with the help of Google, I can find out that

## ****log (1000.001) = 3.00000043****

### All right, one more special case…

## Somebody out there is asking, what about negative numbers? Where do they fit into all of this???

## balance-rollover

## Well, the short answer is, they don't. Negative numbers are the left out in the cold when it comes to logs, which is generally all right. Usually we are using logs because we want to compare the amount of something we counted ... and that's usually a positive number, otherwise we couldn't count it.

# Sometimes it does cause problems (especially in statistics), so people who use logs a lot have ways to get around those problems, but we won't discuss that here. For our purposes, just remember ****you can't take the log of a negative number****, but even if you do, feel sorry for it!

## The log is the power!

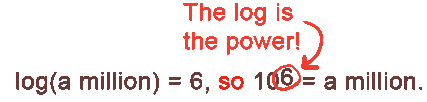
## ****The mathematical definition of a logarithm to the base 10 (log10) is:****

## ****If *x*= 10*y* then *y* = log *x*****

## ****The log10 of a number is the power to which 10 must be raised to give that number.****

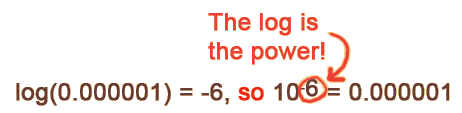
## ****For example 100 = 10 🞨 10 = 102, (this can also be written as 10^2), and so 2 = log (100).****

"Counting the zeros" is the same as saying "what power would I need to raise 10 to in order to get this measurement?" That's important, but hard to remember. The short version of "what power would I need..." is:



**= 1 🞨 10 🞨 10 🞨 10 🞨 10 🞨 10 🞨 10**

Likewise, if your chances of winning the lottery are 1-in-a-million ( = 0.000001), we could say that



**Base Exponent**

**= 1 ÷10 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10**

**The base is 10 and the power is the exponent.**

You can use the applet below to practice finding the log of easy (non-messy) numbers. Keep at it until it becomes very easy!

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| --- | --- |
| The online version of this module has an interactive applet which allows you to practice estimating the logs of biological measurements. To find this applet go to: <http://mathbench.umd.edu/modules/measurement_logs/page04.htm> |  |

## What about other numbers?

So far we have been finding logs of pretty easy numbers -- various arrangements of a '1' and some zeros. So, it's easy to figure out that log (100) = 2 and log (1000) = 3. But of course (being scientists) we know that the numbers we're really interested in aren't going to be so simple.

To what power must 10 be raised in order to obtain the value of 6? The number 6 is between 1 and 10 so the log of 6 must be greater than the log of 1 (i.e., 0) and less than the log of 10 (i.e., 1). The actual value of log (6) is 0.77815.

How about the log of 180? 279? 736.2? Well, first of all we know that all of the logs for these numbers must be between 2 and 3. But where exactly? There is no easy formula for calculating this ... here is a case where you really do need a calculator (or a slide rule, or a lookup table, or enough time to do a little calculus).

**But, you can certainly make an informed ESTIMATE of the log.**

For example, 180 is about a tenth of the way between 100 and 1000. So the log should be somewhere around 2.1. Let's check that out...

**log (180) = 2.26 (We guessed 2.1)**

Well, that's more or less in the ballpark. If you have done any log-transforming of graphs, you know that logs tend to exaggerate small differences. So it makes sense that the log would be a little higher than our original guess.

Let's try another one. 279 is about a third of the way between 100 and 1000. We could guess 2.25, but let's increase that a little ("exaggerate" it). How about 2.35? (Feel free to make your own guess here...)

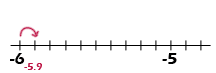
**log(279) = 2.45 (I guessed 2.35)**

Well, as you can see, estimating logs takes a bit of practice... How about 736.2? This is about 2/3 of the way between 100 and 1000, and I'm going to make sure I exaggerate a lot here ... how about 2.85?

**log(736.2) = 2.87 (I guessed 2.85)**

## And what about SMALL numbers

Consider an *E. coli* ... measuring 0.000002 m. It would be easy to say log (0.000001) = −6 ... but *E. coli* are bigger than that. Not big enough to get to a log scale number of −5, but about a tenth of the way between − 6 and − 5. If you imagine a number line, **you'll see that a tenth of the way between −6 and −5 = −5.9**



So if we do some "exaggerating", we might guess −5.8 or even more. In fact,

**log (0.000002) = −5.7 (*E. coli*)**

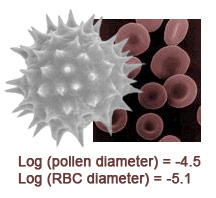
Likewise, red blood cells have a diameter of 0.000007 m, so they are also between - 6 and - 5 on a log scale:

**log (0.000007) = −5.2 (red blood cell)**

You can tell that *E. coli* are smaller than red blood cells, because −5.7 is less than −5.2.

Now that we know that normal red blood cells are approximately −5.2 on a log scale, we can also guess that a blood cell that is a −4 is going to be very problematic for the poor creature whose capillaries get exploded by it. And a red blood cell that's a −6 will be too tiny to do any good.

**A red blood cell diameter is −5.2 on the log scale, while a typical pollen grain is a −4.5. Which is smaller?**



* I need a hint : The more negative the log, the smaller the measurement.
* another hint : The log of red blood cell diameter is more negative than the log of pollen grain diameter, so it must be smaller

I think I have the answer: **The red blood cell is smaller.**

## Your turn with other numbers

This time, try to guess the log scale value of some real (biologically important) measurements. Remember, it’s really not possible to calculate the logs exactly (not for many numbers, anyway, and those are the numbers we care about in science). You're just trying to get an intuitive “feel” for how logs correspond to numbers in the real world...

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| --- | --- |
| The online version of this module has an interactive applet which allows you to practice estimating the logs of biological measurements. To find this applet go to: <http://mathbench.umd.edu/modules/measurement_logs/page07.htm> | messy_numbers_module |

Here is a link to a really cool website which shows the scale of the universe from the really, really small to really, really big. It is definitely worth a look: [http://htwins.net/scale2/](http://htwins.net/scale2/" \t "_blank)

## Anti-Logs

By the way, if you know the log value and you want to find the original value of the number, how do you do it?

Use the anti-log, of course!! The anti-logarithm (antilog) is the number corresponding to a given log, for example, the log of 1000 is 3, thus 1000 is the anti-log of 3. To get the anti-log on **your calculator, you can raise 10 to the log, or use the 10*x* key.**

For example, the log of the pollen diameter was −4.5, therefore the diameter in metres is

**10−4.5 ≈ 0.0000316 m**

**Now,**

**0.0000316 = 3.16🞨10 -5 m or 31.6 µm**

If you are using Google as a calculator, type "10^ −4.5 = " and hit ENTER.

**How big is a fly's eye? The log of the diameter is −3.1...**

* I need a hint: Remember, the log is the exponent.
* another hint: Raise 10 to the log to get the original measurement back.
* another hint: Use the 10x key on your calculator, or type "10^−3.1 = " on Google.

I think I have the answer: **10−3.1 = 0.00080 m.**

You can estimate: If the antilog is 3.5 then the number must be between 1000 (103) and 10.000 (104)

103.5 = 3162

What is the antilog of -0.8?

Anti-log (-0.8) = 10-0.8 = 6.31

## Measuring the power of an earthquake

Any time that a measurement can vary over many orders of magnitude, that's a candidate for using a log scale.

Here's another example you probably won't see in your biology class (unless you get your tutor really mad, maybe), but you've probably heard of it. Earthquakes are measured on a scale of 0 to 9, corresponding to how much energy they release. This is a log scale, so each whole number on the Richter scale represents a ten-fold increase in energy.

**An earthquake hit Christchurch in New Zealand, on 22 Feb 2011 measuring 6.3 on the Richter scale (tragically 185 people were killed). In 1929 an earthquake in Murchison NZ measured 7.3, leaving 17 dead. How much more energy was released in the Murchison earthquake?**

* I need a hint… > Each whole number on the Richter scale multiplies the linear measurement of energy by 10.

Answer: **101 = anti-log 1 = 10 times as much**

In the example above, the logged numbers were exactly 1 unit apart, and the quick answer to how much more energy was released would be simply

**101 = 10 times as much.**

Likewise, if the earthquakes measured 2 and 6 on the Richter scale, that's a difference of 4 points, and the difference in actual energy is

**104 = 10,000 times as much.**

Of course the logged number you encounter will not usually increase by exactly 1 unit. In the real world, measurements are messier. But the principle is the same: **find the difference between the two logged numbers, and "unlog" that difference.**

**The earthquake in Newcastle Australia in 1989 measured 5.6 on the Richter scale, How much more energy was released in the Christchurch earthquake vs. the Newcastle earthquake?**

* I need a hint… > The difference between the two earthquakes was 0.7 on the Richter scale (6.3 -5.6).
* Another hint… > The anti-log of 0.7 is 100.7

10^0.7 = **5 times more energy**

## Measuring Acidity: the pH scale.

The proper function of biological systems depends on the correct concentration of hydrogen ions (H+ or more correctly H3O+) in an aqueous solution. The concentration of H+ in water can be as high as 1.0 M (extremely acidic), or as low as 0.00000000000001 or 10-14 M (extremely basic). Instead of counting out the zeros every time, we use a log scale. The pH scale was devised by Soren Sorenson to simplify dealing with the wide range of hydrogen ion concentrations in aqueous solutions. The pH is defined as the negative logarithm of the hydrogen ion concentration:

pH = - log[H+] and [H+] = 10 -pH

**extremely acidic : [H+] =1 M (100): pH = 0**

**extremely basic : [H+] = 10-14 M: pH = 14**

The concentration of H+ in pure water is 10-7M, as is the concentration of OH- ions. H2O is itself a weak acid and dissociates into H+ and OH -. The equilibrium constant for this dissociation is:

K = [H+ ] [OH-]

[H2O]

The value of K is 1.8 🞨 10-16 M at 25 0C. This can be simplified by realizing that there is a large excess of water which has a molar concentration of 55.56 M and can be assumed to be constant. The equation simplifies to

K [H2O] = [H+] [OH-] = 1.8 🞨 10-16 🞨 55.56 = 1 🞨 10 -14 M2.

This is known as the ionic product of water Kw. Kw = [H+] [OH-] = 10-7 🞨 10-7 = 10 -14 M2

If the pH of the solution is less than 7 it means that the [H+] is greater than 10-7 M and the solution is acidic. If the pH is greater than 7, [H+] is less than 10-7 M and the solution is basic. A solution of pH 7 is neutral.

The space below has room for 100,000 pixels. If the H+ concentration is 0.0055 M (5.56 🞨 10-3 M), it means 1 out of 1000 spaces should be filled by a dot representing a hydrogen ion. So what you're currently seeing is a visual representation of a pH of 2.25 (still pretty highly acidic). (Remember the molarity of pure water is 55.56M). Clicking on the buttons will show you the visual representation of pH 1 (super acidic) through 5 (still slightly acidic).

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| --- | --- |
| The online version of this module has an interactive applet which allows you to practice visualizing pH. To find this applet go to: <http://mathbench.umd.edu/modules/measurement_logs/page09.htm> | visualizing_ph_module |

*As a general rule, when you add 1 to a log number, it’s the same as multiplying the unlogged measurement by 10:*

**1 on a log scale corresponds to 10 on a linear scale**

**2 on a log scale corresponds to 100 on a linear scale**

**3 on a log scale corresponds to 1000 on a linear scale**

**and so on...**

It works a little differently for pH, because pH is the negative log of concentration. So, **every time we SUBTRACT a single pH unit (like going from 2 to 1), we multiply H+ by 10.**

Unfortunately, we can't get any less acidic than pH = 5 on the applet above, because we'd have to make the screen huge!! To get a pH of 14, we'd need the screen to be more than 10,000 times taller and 10,000 times wider, and this humongous screen would contain only a single dot.

Question: If the pH is increased from 4 to 5, what is the change in [H+] concentration?

Answer: 10 times less.

## Here are two problems

**The pH of lemon juice is 4.2. The pH of milk is 7.8. What is the difference in hydrogen ion concentration (approximately)?**

* I need a hint… > The difference in pH is 7.8 − 4.2 = 3.6
* Another hint… > So the difference in concentration will be somewhere between "3 zeros" and "4 zeros" (between 1000 and 10000).
* Another hint… > 103.6 = 3981.
* Another hint… > Make sure you know which one is more concentrated!
* Another hint… > Lemon juice is more acidic, so it should have more H+. Therefore, lemon juice has 10 3.6 = 3981 times as many hydrogen ions as milk.

Answer > **3981 times as much.**

Finally, can you figure out how to use the "unlogging" trick to calculate the concentration?

**The pH of lemon juice is 4.2. How many moles of H+ are present in 5 litres of lemon juice?**

* I need a hint… > Use "the log is the power" to figure out the amount of H+ in 1 litre of lemon juice (and don't forget that it is really MINUS 4.2...)
* Another hint… > If you forgot the minus sign, you end up with 16,000 moles of H+ in one litre of lemon juice, which is obviously wrong! ... Then you remember the minus sign.
* Another hint… > So how much of H+ in five litres?
* Another hint… > 10−4.2 = 0.000063 moles, so 5 litres of lemon juice contains 5 times as much -- 0.000315 moles of H+.

Answer > **5🞨10−4.2 = 0.000315 moles of H+**

## Summary

Logs make it easier **to compare measurements that vary by many orders of magnitude.**

**Positive logs** mean big numbers – bigger than one.   
To find the approximate log, simply **count the number of digits AFTER the first digit.**

**Negative logs** mean small numbers – between zero and one.   
To find the approximate log, **count the decimal point plus the number of zeros** UNTIL the first non-zero digit.

Logs are the same as the exponent you would need to put on a "10" in order to get your original measurement: in other words, **The Log is the Power.**

Going UP BY ONE on a log scale is always the same as multiplying by 10. Going DOWN BY ONE on a log scale is always the same as dividing by 10.

You can **recover the original measurement** by raising 10 to the log ( or "10^\_\_\_ =" on Google).

The **pH scale** is based on the NEGATIVE log of the concentration of H+ ions.

The **Richter scale** is based on the log of energy released.

## Learning Outcomes

After completing this module you should be able to:

* Work with very large and very small numbers by converting between log and linear scales, using logarithms to the base 10 and anti-logarithms
* Use the log scale of hydrogen ion concentration (pH) to determine the acidity of the solution
* Determine the hydrogen ion concentration of a solution from its pH